Hong Kong Mathematics Olympiad (2024/25)

Finals (Group – Event 1) (Modified by EDB)

FOR OFFICIAL USE

Score for accuracy		$\times$	Mult. factor for speed		$=$		School ID	**FE-**
			$+$	Bonus Score			Time	
				Total Score			Min.	Sec.

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特别声明，答案须用数字表达，并化至最简。

1. Let x be a real number such that $\sin^{10}x+\cos^{10}x=\frac{1}{16}$. If $\sin^{12}x+\cos^{12}x=\frac{p}{q}$, where p and q are relatively prime positive integers. Find the value of $\frac{p}{q}$.
设 x 为一个实数，使得 $\sin^{10}x+\cos^{10}x=\frac{1}{16}$。若 $\sin^{12}x+\cos^{12}x=\frac{p}{q}$，其中 p 和 q 是互质的正整数。求 $\frac{p}{q}$ 的值。

2. Find the least value of x that satisfies the equation $\sqrt{x+3-4\sqrt{x-1}}+\sqrt{x+8-6\sqrt{x-1}}=1$.
求满足方程 $\sqrt{x+3-4\sqrt{x-1}}+\sqrt{x+8-6\sqrt{x-1}}=1$ 的 x 的最小值。

3. In ΔABC, $AB=5$, $BC=6$ and $AC=7$. A point D is chosen inside the triangle (excluding the boundary points) such that the areas of triangles ABD, BCD and CAD are equal. Find the length of AD.
在 ΔABC 中，$AB=5$，$BC=6$ 及 $AC=7$。在三角形内选取一点 D（不包括边界点），使得三角形 ABD、BCD 和 CAD 的面积相等。求 AD 的长度。

4. How many prime numbers p are there such that p^2+2 is also a prime number.
有多少个质数 p 使得 p^2+2 也是质数？

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Finals (Group – Event 2) (Modified by EDB)

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Score for accuracy		×	Mult. factor for speed		=		School ID	**FE-**	
			+	Bonus Score			Time		
				Total Score				Min.	Sec.

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1. Let a, b, c, d be real numbers satisfying the following equations

$$\begin{aligned} a+\ \ b+\ \ c+\ \ d &= 3 \\ a^2+2b^2+3c^2+6d^2 &= 5 \end{aligned}$$

Let α be the maximum value of a. Find α.
设 a, b, c, d 是满足下列方程的实数

$$\begin{aligned} a+\ \ b+\ \ c+\ \ d &= 3 \\ a^2+2b^2+3c^2+6d^2 &= 5 \end{aligned}$$

设 α 为 a 的最大值。 求 α。

2. Let β be the number of integer solutions of the equation $x^2-12x+y^2+2=0$. Find β.

设 β 为方程 $x^2-12x+y^2+2=0$ 的整数解的数量，求 β。

3. Given that $\gamma=(\cot 10°-4\cos 10°)^2$, find γ.
已知 $\gamma=(\cot 10°-4\cos 10°)^2$，求 γ。

4. Let $c\in(0,1)$ and f be a function on $[0, 1]$. Suppose that $f(0)=0, f(1)=1$ and for any $x\le y$

$$f\left(\frac{x+y}{2}\right)=(1-c)\,f(x)+cf(y).$$

Find $f\left(\dfrac{c}{5c+1}\right)$.
设 $c\in(0,1)$，f 为 $[0, 1]$ 上的函数。假设 $f(0)=0$，$f(1)=1$ 且对任意 $x\le y$

$$f\left(\frac{x+y}{2}\right)=(1-c)\,f(x)+cf(y)\text{。}$$

求 $f\left(\dfrac{c}{5c+1}\right)$。

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Finals (Group – Event 3) (Modified by EDB)

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1. Let x be a positive integer with three digits, when $2x$ is divided by 3, the remainder is 1; when $3x$ is divided by 5, the remainder is 3; and when $5x$ is divided by 7, the remainder is 1. Find the number of all possible values of x.
设 x 为一个三位正整数，当 $2x$ 除以 3 时，余数是 1；当 $3x$ 除以 5 时，余数是 3；当 $5x$ 除以 7 时，余数是 1。求 x 所有可能值的数量。

2. Suppose that when rolling a six-sided die (each with a number from 1, 2, 3, 4, 5, or 6), the probability of getting each number is equal. If two such dice are rolled simultaneously in a game, the probability of each rolling a number from 1, 2, 3, 4, 5, or 6 is also equal. Assuming that the two numbers are different, find the probability that one of them is 6.
假设一颗有 6 个面的骰子（每颗骰子上的数字分别为 1、2、3、4、5 或 6），掷出每个数字的概率相等。如果在游戏中同时掷出两颗这样的骰子，每颗骰子掷出 1、2、3、4、5 或 6 的机率也相等。假设两个数字不同，求其中一个数字为 6 的概率。

3. Given the equation $2x_1 + x_2 + x_3 + x_4 + x_5 = 3$. Find the number of non-negative integer solutions of it.

给定方程式 $2x_1 + x_2 + x_3 + x_4 + x_5 = 3$。 求它的非负整数解的数量。

4. Given that x is a positive real number and $x = \sqrt{x - \frac{1}{x}} + \sqrt{1 - \frac{1}{x}}$. Find x.

假设 x 是一个正实数，且 $x = \sqrt{x - \frac{1}{x}} + \sqrt{1 - \frac{1}{x}}$ 。求 x。

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Finals (Group – Event 4) (Modified by EDB)

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1. If $x+y+z=15$, how many distinct combinations of positive integer roots exist?
若 $x+y+z=15$，有多少组不同的正整数解存在？

2. Find the value of $\left(\sqrt{7}+\sqrt{3}+\sqrt{2}\right)\left(\sqrt{7}+\sqrt{3}-\sqrt{2}\right)\left(\sqrt{7}-\sqrt{3}+\sqrt{2}\right)\left(-\sqrt{7}+\sqrt{3}+\sqrt{2}\right)$.
求 $\left(\sqrt{7}+\sqrt{3}+\sqrt{2}\right)\left(\sqrt{7}+\sqrt{3}-\sqrt{2}\right)\left(\sqrt{7}-\sqrt{3}+\sqrt{2}\right)\left(-\sqrt{7}+\sqrt{3}+\sqrt{2}\right)$ 的值。

3. If α and β are the real roots of the equation $x^2-2mx+m+6=0$, find the minimum value of $(\alpha-2)^2+(\beta-2)^2$.
若 α 和 β 是方程 $x^2-2mx+m+6=0$ 的实根，求 $(\alpha-2)^2+(\beta-2)^2$ 的最小值。

4. Find the remainder when $1^3+2^3+3^3+\cdots+2025^3$ is divided by 7.
求 $1^3+2^3+3^3+\cdots+2025^3$ 除以 7 的余数。